Stochastic modeling to reflect investment risk in funding liabilities for pension plans

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Introduction

With the launch of Actuarial Standard of Practice No. 51, *Assessment and Disclosure of Risk Associated with Measuring Pension Obligations and Determining Pension Plan Contributions*, retirement actuaries have an opportunity to help employers better understand the risks associated with their pension plans, which could lead to better decision making when managing these plans.

Making good decisions often involves making reasonable projections about possible outcomes. As with many decisions, some of these possible outcomes may be desirable while other outcomes may be undesirable. Some of the possible outcomes may be likely while other outcomes are unlikely. This simple and intuitive way to understand risk reflects how many of us make big life decisions. This is the understanding we should be presenting to employers as we explain plan-related risks.

Until now, explicit disclosures about plan-specific risks have rarely been included in funding valuation reports. While ASOP 51 has changed this, it would not be surprising if some employers find it difficult to grasp their plans' risks from non-numerical assessments and plan-maturity measures.

By tweaking existing familiar concepts—the funding liabilities—we can leverage the understanding that employers already have about their pension plans to explain various risks, some of which are very pertinent to plan decision making. This tweak also allows us to express risks in the simple, intuitive way suggested earlier—some possible outcomes may be more or less desirable, and those same outcomes may be more or less likely.

Weaknesses of current measurements of funding liabilities

Funding liabilities generally means the actuarial present value of benefits and the actuarial accrued liability measured with actuarial assumptions and methods selected for contribution budgeting. This does not mean the funding target or current liability as defined by sections 430 and 431 of the Internal Revenue Code, but instead means liabilities typically used to determine actuarial contributions for public pension plans. It also means an actuarial accrued liability used for private plans when determined on a first principles basis, generally with assumptions and methods identical or similar to those used under FASB ASC Topic 960.

While many assumptions are important, one of the more important ones is the rate of return on assets, which retirement actuaries often use as the discount rate in funding valuations as applicable law permits.

In my experience, actuaries often consider investment yield assumptions carefully and reflect both the expected annual return and the effect that volatility has on the long-term rate of return. However, our method of measuring liabilities with a single rate or a single yield curve has some weaknesses that I aim to change:

- The use of a single rate or a single yield curve may make the rate-of-return assumption resistant to change, especially if the employer must approve any changes to the assumption.
- The expected yield on plan assets from the valuation date depends on the time horizon, and it isn't always clear what time horizon should be used to select the assumption for expected yield on plan assets when a single rate is used.
- It isn't immediately clear to the employer what effects the selection of plan assets has on the soundness of the pension plan.
- The use of a single assumption produces a single value for each type of funding liability. Only a theoretical average is produced.
  - This gives no indication of how desirable or likely various outcomes could be.
  - Too much emphasis becomes placed on a single liability as a funding standard—that theoretical average. Employers could instead set their funding standards to meet specific risk targets for funding their pension plans.
- A single value for each funding liability can lead some actuaries to provide unreasonable or misleading information to employers. An example would be reporting the actuarial present value of future employer contributions as zero merely because the assets slightly exceed the actuarial present value of benefits—that theoretical average.

Route to improvement

To tweak the funding liabilities to overcome some of these limitations, it is best to start with the actuarial present value of benefits. To clarify both my use of language and the general framework, I will define actuarial present value of benefits (APVB) as the amount of assets that if held on the measurement date, would be both necessary and sufficient to pay all future benefits to current participants as of the measurement date on average. I will use present value of benefits (PVB) to mean the same thing as APVB except the words *on average* are dropped from the definition. Under this framework, the APVB is the expected value of all possible PVBs, which have a distribution that we will attempt to model stochastically.

The actuarial present value of benefits is generally calculated using the following equations:

(1) $APVB = \sum_t E\,[B_t|Y_t]\;E\,[(1+Y_t)^{-t}]$

or

(2) $APVB = \sum_t E\,[B_t]\;E\,[(1+Y_t)^{-t})]$ when B_t and Y_t are independent

In this equation, t is the amount of time (in years) after the measurement date, and $E[x]$ is the expected value of x. B_t is the total benefits payable at time t, and Y_t is the assumed annual yield on assets at time t. B_t and Y_t are random variables that can be modeled stochastically. Stochastic modeling B_t is beyond the scope of this paper, but B_t and Y_t may not be independent (consider salary-related plans or plans with cost-of-living adjustments [COLAs]). Actuaries should consider this possible dependence when applying the methods laid out here. The model set forth here focuses on stochastically modeling Y_t to estimate the distribution of the present value of benefits.

While Y_t can be modeled in several ways, one way would be to assume that annual returns on assets are normally distributed and independent from year to year. Although there have been criticisms of this particular model for returns on assets[1], the point is less about picking the best model for Y_t and more about giving a general idea of how modeling Y_t can improve communications about risks to employers and pension plan sponsors.

In the appendix, I describe a method to model Y_t and the present value of benefits stochastically under the two assumptions. The first assumption is that annual rates of return are independent and normally distributed, and the second is that benefit payments are independent of annual rates of return. The result of this method is a series of N trials, each giving a simulated value for the present value of benefits. And when these simulated values are averaged together, they approximate the actuarial present value of benefits.

1 Adcock, C.J. & Shutes, K. (2000) Fat Tails and Capital Asset Pricing Model. In: Dunis C.L. (eds) Advances in Quantitative Asset Management. Studies in Computational Finance, vol 1. Springer, Boston, MA

The expression for the PVBs and the APVB are given by the equations below:

(3) $PVB_n = \sum_t E\,[B_t]\;(1+Y_{t,n})^{-t}$

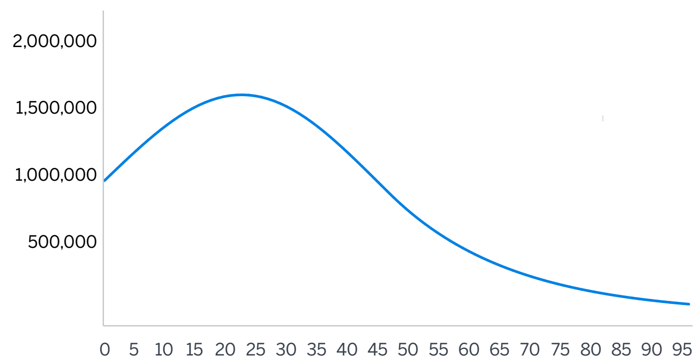
(4) $APVB = \frac{1}{N}\sum_n \sum_t E\,[B_t]\;(1+Y_{t,n})^{-t}$

In these equations, N is the number of trials, n is an index that points to given trial, and t is the amount of time (in years) after the measurement date. $E[x]$ is the expected value of x. B_t is the total benefits payable at time t, and Y_t is the assumed annual yield on assets at time t.

Example

To make this more concrete, a hypothetical example is shown below. The following line graph summarizes the expected benefit payments for a hypothetical plan. These expected benefit payments were selected with the hope of representing a typical plan.

FIGURE 1: SAMPLE EXPECTED BENEFIT PAYMENTS



The assets supporting this hypothetical plan are invested in a portfolio with 60% in equities and 40% in fixed income with 6.20% as the expected annual return on assets in any year and 10.40% as the expected standard deviation of annual returns. The expected long-term yield on assets is about 5.17%.

With a 5.17% discount rate, the actuarial present value of benefits is $25,133,415. Using a million-trial Monte Carlo simulation, the actuarial present value of benefits is $25,100,789, about 0.1% less than the single-rate present value of benefits.

However, with the additional information from the trials, there are estimates of various percentiles for the present value of benefits.

The actuarial present value of benefits answers the question of what value of assets (on average) would be necessary to pay benefits earned or expected to be earned by participants as of the measurement date. The present value of benefits at a given security threshold can answer a similar question. What value of assets would be necessary such that the employer should feel X percent certain the assets would cover all benefits earned or expected to be earned by participants (given that benefit payments are known)?

FIGURE 2: PRESENT VALUE OF BENEFITS AT SELECTED SECURITY THRESHOLDS

SECURITY THRESHOLD	PRESENT VALUE OF BENEFITS
5TH PERCENTILE	$15,035,051
10TH PERCENTILE	16,492,124
25TH PERCENTILE	19,417,862
50TH PERCENTILE	23,586,742
75TH PERCENTILE	29,060,991
90TH PERCENTILE	35,520,401
95TH PERCENTILE	40,285,430
AVERAGE	25,100,789

The stochastic model for the present value of benefits allows us to explain how the riskiness of assets affects the health of the plan. For example, consider the same plan supported with a less risky asset allocation, 40% in equities and 60% in fixed income with 5.60% as the expected annual return on assets in any year and 7.40% as the expected standard deviation of annual returns. The expected long-term yield on assets is about 5.08%.

With a 5.08% discount rate, the actuarial present value of benefits is $25,503,435. The Monte Carlo simulation gives an actuarial present value of benefits of $25,473,321, about 0.1% less than the single-rate present value of benefits. The actuarial present value of benefits using the riskier asset allocation is indeed smaller, but the percentiles for the present value of benefits tell a more complete story.

FIGURE 3: PRESENT VALUE OF BENEFITS BY PORTFOLIO AND SELECTED SECURITY THRESHOLDS

SECURITY THRESHOLD	PVB USING RISKIER PORTFOLIO	PVB USING LESS RISKY PORTFOLIO
5TH PERCENTILE	$15,035,051	$17,677,827
10TH PERCENTILE	16,492,124	18,965,105
25TH PERCENTILE	19,417,862	21,413,068
50TH PERCENTILE	23,586,742	24,670,571
75TH PERCENTILE	29,060,991	28,643,787
90TH PERCENTILE	35,520,401	32,980,588
95TH PERCENTILE	40,285,430	35,969,432
AVERAGE	25,100,789	25,473,321

The plan's actuarial present value of benefits valued with the less risky assumptions is only about 1.48% more than when valued with the riskier assumptions, but the present value of benefits is 7.15% less at the 90th security level and almost 11% less at the 95th security level. Conversely, the PVB estimated with the assumptions consistent with the less risk portfolio is higher at lower security levels. This would suggest that having less risky assets at higher levels of funding may be better for this plan. This relationship can help encourage employers to adopt liability-driven investment (LDI) strategies.

The present value of benefits at various security levels can give employers a better sense of both the likelihood and the amount of future funding needed for their current plan participants. It also can offer an idea about how to adjust the plan's asset allocation to manage some forms of investment risks.

Other funding liabilities

This paper has mostly discussed the present value of benefits, but this method can work for some other funding liability measures. Besides the present value of benefits, it most easily works with the actuarial accrued liability (AAL) under projected unit credit and traditional unit credit funding methods. For the unit credit funding method, the actuarial accrued liability can be calculated using the same methods used for the actuarial present value of benefit by just substituting the expected benefit payments with the accrued benefit payments (with or without salary increases). The normal cost benefit payments can be used to calculate the total normal cost.

Stochastic actuarial accrued liabilities would be computationally intensive for funding methods other than unit credit methods.

Funding standard

Retirement actuaries often recommend contributions that work toward funding on an expected-value basis. While this may be good for private plans where the employer may bear a cost for overfunding (large excise taxes on assets reverted to the employer), the advantages of this method are less clear for public plans.

With a stochastic present value of benefits given at various security levels, we can consider raising the funding standard for public pension plans beyond an expected-value basis toward a likely-to-succeed basis. Instead of funding to a level with about a 40% chance of becoming insolvent if no future contributions are made, perhaps we can show employers that funding to a level with only a 25% chance of insolvency without future contributions (i.e., the 75th security level) may better fit their needs and risk management strategies.

Conclusion

This paper presents the idea of stochastically modeling funding liabilities with the aim of helping fellow retirement actuaries talk with pension plan sponsors about risks in meaningful ways (and comply with ASOP 51). It's more natural and approachable to view funding risks in terms of how much money the plan might need rather than how might plan assets perform. While both views are important to understand pension risks, the former can provide a useful perspective that is rarely discussed or shared directly with employers.

This said, I invite all retirement actuaries who read this to consider, improve, or criticize the model.

CONTACT

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Appendix – Stochastic Model of the Present Value of Benefits

As a reminder, the actuarial present value of benefits is generally calculated using the following equation when benefit payments and returns on assets are independent:

(1) $APVB = \sum_t E[B_t]E[(1 + Y_t)^{-t}]$

In this equation, t is the amount of time (in years) after the measurement date, and $E[x]$ is the expected value of x. B_t is the total benefits payable at time t, and Y_t is the assumed annual yield on assets at time t. Y_t can be modeled in several ways, but here we will assume that annual returns on assets are normally distributed and independent from year to year. B_t will be assumed to be constant.

Let's set up the model of Y_t in terms a different random variable R_t, which is the annual rate of return for the year ending at time t.[2] Because we intend to estimate present values of benefits as presented in expression (1), it would be useful to model the discount factor $(1+Y_t)^{-t}$ for each time t in addition to Y_t. The discount factor at any time T is the product of the discount factors for individual one-year periods from $t = 0$ to $t = T$. Each one-year discount factor can be expressed as $(1+R_t)^{-1}$, similar to $(1+Y_T)^{-T}$. This gives the following equations relating Y_T and R_t:

(2) $(1 + Y_T)^{-T} = \prod_{t=0}^{T}(1 + R_t)^{-1}$

(3) $Y_T = [\prod_{t=0}^{T}(1 + R_t)^{-1}]^{-1/T} - 1$

Because we often calculate benefit payments for each year beginning on the valuation date and discount those benefit payments from the middle of each valuation year, it would be convenient to rewrite the equations above to give yields at the midpoint of the valuation years. This would be where t takes on the values in the set $S_T = \{0.5, 1.5, 2.5, 3.5, \ldots, T\}$:

(4) $(1 + Y_T)^{-T} = \prod_{t \in S_T}(1 + R_t)^{-\min(1,t)}$

(5) $Y_T = [\prod_{t \in S_T}(1 + R_t)^{-\min(1,t)}]^{-1/T} - 1$

We can estimate $E[(1+Y_T)^{-T}]$ and the actuarial present value of benefits by using Monte Carlo simulation. To complete the simulation with N trials, we first generate N uniformly distributed random variables for each time $t \in S_T$ that we care about. Next, using the expected annual return and expected standard deviation of the annual return of the supporting asset portfolio based on capital market assumptions, convert those uniform random variables into normally distributed simulated returns. This conversion would be calculated as:

(6) $R_t = \phi^{-1}(U_t)\sigma\sqrt{\min(t,1)} + \mu$

Here U_t is the uniform random variable for the period ending at time t, μ is the expected annual rate of return in a single year, σ is the expected annual standard deviation, and $\emptyset^{-1}$ is the inverse of the standard normal cumulative distribution function.

Once the simulated rates of return are calculated for each trial and period, we can calculate the yield at each time of concern as given in equation (5). If the expected value of the discount factor $E[(1+Y_t)^{-t}]$ is replaced with the discount factors generated from a single trial, the result is the calculation of that single trial's PVB, as shown in equation (7) below.

(7) $PVB_n = \sum_t E[B_t](1 + Y_{t,n})^{-t}$

The average of the PVBs given from all trials is the actuarial present value of benefits.

(8) $APVB = \frac{1}{N}\sum_n \sum_t E[B_t](1 + Y_{t,n})^{-t}$

Here $Y_{t,n}$ is the annualized yield at time t under the nth trial.

If one adjusts the projected benefit payments to account for the possible correlation between those payments and the yield, equations (7) and (8) become equations (9) and (10) below.

(9) $PVB_n = \sum_t E[B_{t,n}|Y_{t,n}](1 + Y_{t,n})^{-t}$

(10) $APVB = \frac{1}{N}\sum_n \sum_t E[B_{t,n}|Y_{t,n}](1 + Y_{t,n})^{-t}$

2 If t is less than 1, let R_t be the annualized rate of return from time zero to time t.