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Practical Considerations in Variable Annuity Pricing





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INTRODUCTION

The variable annuity (VA) was first introduced in the United States by the Teachers Insurance and Annuities Association - College Retirement Equity Fund (TIAA-CREF) in 1952. Sales have grown since then, stimulated by offering tax-deferred savings benefits and also guaranteed living benefits (GLB) and guaranteed minimum death benefits (GMDB). The sales peaked in 2007, with total sales towering above US\$180 billion. After the financial crisis, sales in 2008 were 15% below those in 2007 and sales in 2009 were another 20% lower.

In addition to the low sales in 2009, the VA industry also went through probably the biggest de-risking and reshuffling since its debut. What once was the competition for the richest GLB took a 180-degree turn to a flight for safer products. Guarantees were lowered, charges were increased, and more stringent investment restrictions were introduced. Several major VA writers conceded their leading positions in sales by announcing a large-scale retreat from the VA market. The problem the VA industry had during the financial crisis was multi-faceted, but a common lesson the industry learned was that VA products could be risky, probably much more risky than had been recognized before.

However, 2010 witnessed a slow recovery where the sales of VA up to the third calendar quarter bounced back by 7% from the same quarter to date in 2009. Some relatively rich products, e.g., with high roll-up rates, found their way back to the market even though the charges still stayed high. It might be premature to announce that a new wave of competition for rich VA products has started. But with sales of fixed products struggling in a low-interest environment, it may not be unreasonable to expect the sales of VA to continue their growth in 2011.

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It is thus timely for actuaries working in the VA design and pricing area to review the pricing process and consider what might have gone amiss prior to the financial crisis. We hope that the views we present in this paper will help guide actuaries through their review processes. In particular, we will discuss the important issues actuaries need to consider in a VA pricing process. These considerations include those from the broad methodological perspective and those from the specific modeling perspective.

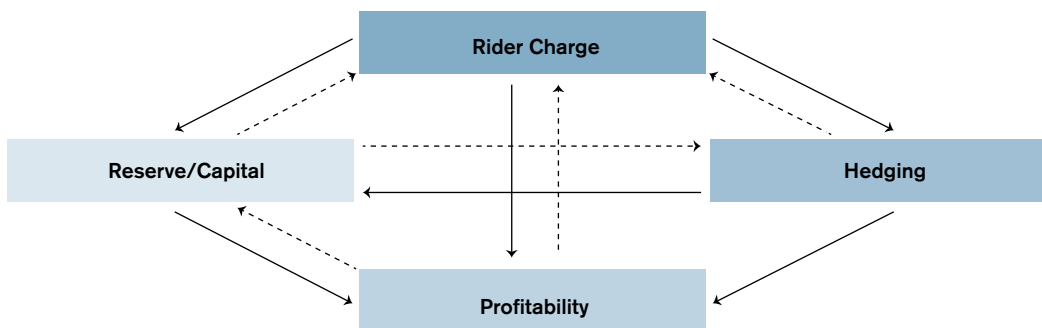
From the broad perspective, the VA pricing methodology is a search for an optimal balance between risk and return. The search involves considerations and analyses of four broadly intertwined elements. Actuaries need to fully understand, analyze, and communicate to senior management the interaction of these elements. The ultimate decision will be based on each company's risk appetite. We will discuss these elements in detail in Sections II-VI. Section II provides an overview of the four key elements in the pricing process, with each element discussed in more detail in Sections III-VI. We will then close with the summary in Section VII, which also discusses some other important considerations in the VA pricing process.

From the specific modeling perspective, the analyses of the four elements present some technical modeling challenges in a VA pricing exercise. We will discuss some of these challenges through a case study in the Appendix.

SECTION II: THE FOUR ELEMENTS

A traditional product pricing exercise searches for the premium rate that meets the profit target. Despite the same fundamental objective, a VA pricing exercise is much more complicated because of the presence of the GLB. Its premium rate is in the form of an asset-based charge, which is at the same time an additional deduction to the account value growth and thus increases the cost of the guarantee. Its reserve and capital requirements are risk-based and not easy to determine. Its profitability is extremely sensitive to market conditions and policyholder behavior, and thus warrants an analysis of not only the expected value but also the tail events. Hedging is typically implemented to reduce the economic risk, and directly affects the reserve/capital requirements as well as the profitability. In essence, Figure 1 illustrates the interaction of the key elements in a VA pricing process. The four boxes show the four elements, the arrows indicate a direct impact from one element to the other, and a dashed arrow indicates a potential impact from one to the other.

FIGURE 1: THE KEY ELEMENTS IN A VA PRICING



As we will discuss in more detail in the following sections, the four elements of the pricing process are intertwined and influence each other. It is therefore imperative that the pricing exercise analyzes the interaction of the four elements, which would produce different risk-return profiles with different possible combinations. It is the responsibility of the actuaries to present these combinations and their results in a complete menu for the senior management to study and on which to eventually make a decision. Because of the different risk appetites, different companies would very likely land on different decisions even though they might have the same product under the microscope. Therefore, the VA pricing exercise is essentially a search for the optimal risk-return balance based on each company's risk policy and preference.

The four elements of the pricing process are intertwined and influence each other.

The table in Figure 2 presents an example of the pricing menu related to a VA pricing.

FIGURE 2: VA PRICING MENU EXAMPLE

HEDGING DECISIONS	GLB CHARGE	REDUCTION IN CAPITAL/RESERVE	PROFIT MEASURES (ROA/IRR, ETC.)
NO HEDGING	X1%	Y1%	AVERAGE AND PERCENTILE
DELTA	X2%	Y2%	AVERAGE AND PERCENTILE
DELTA/RHO	X3%	Y3%	AVERAGE AND PERCENTILE
DELTA/RHO/VEGA	X4%	Y4%	AVERAGE AND PERCENTILE

The *Hedging Decisions* column shows some of the typical selections involved in a hedging strategy formulation, the Greek(s) to hedge. Each Greek measures the change of liability value corresponding to a change in an economic factor, and the cost of hedging increases as more Greeks are hedged. In reality, however, hedging decisions involve far more factors, and therefore the list in this column may vary based on the exact hedging considerations faced by the management. The *No hedging* row provides a baseline reference to illustrate the relative changes resulting from other hedging decisions, and may also be a possible selection for management.

The *GLB Charge* column shows the potentially different fees charged to policyholders. In reality, though, competition will usually dictate the desired rider charge before others are decided and it is possible that the column contains the same intended charges for all selections.

The *Reduction in Capital/Reserve* column shows the impact of the pair of hedging decisions and GLB charges on the reserve and capital requirement. The presentation of this column could be either single percentages showing the impact on the initial capital and reserve, or tables of percentages showing the impact on capital and reserves at different guarantee-to-account value ratios and policy durations. It should be noted that not all hedging decisions necessarily produce a reduction in the reserve/capital, which makes it more important that the impact of hedging be studied during the pricing stage.

The *Profit Measures* column shows the financial results from the combination of the hedging decisions, the GLB charge, and the reserve/capital requirement. The measures could be anything that companies typically look at for product approval, but it is important that both the expected value and the distribution (with a focus on left tail) be studied.

**Return is meaningless
unless adjusted for the
risk undertaken.**

The concept behind the pricing menu is not new: The return is meaningless unless adjusted for the risk undertaken. In reality, however, the extent to which actuaries implement this concept in the VA pricing exercise varies widely. Also, a comprehensive analysis presents quite a few technical challenges, which may discourage actuaries from completing the entire process. In the following sections, we will discuss each of the four elements of the process in more detail and also the related technical challenges.

SECTION III: EVALUATION OF RIDER CHARGE

As with any form of insurance premium, the GLB charge should be set at a level to cover the expected cost of the claims under the rider guarantee profit margin. The expected cost of the claims, however, is not as easily calculated as for traditional life insurance products. Simply projecting future cash flows assuming a best-estimate set of assumptions will probably not even result in any projected claims.

A GLB acts like a put option. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price. In the case of a GLB, the policyholder has the right to *sell* the underlying account values, sometimes at a value of zero, prior to the expiry of the GLB rider in exchange for either a guaranteed lump sum or a stream of guaranteed annuity payments. The determination of a put option price is complicated but has been demystified since the introduction of the Black-Scholes option pricing formula and the concept of risk-neutral valuation back in the 1970s. However, its application in VA pricing had a slow start. It was not uncommon that the riders introduced in the VA market prior to the year 2000 and perhaps even in the first few years of the new century were not evaluated using the risk-neutral measure.

It is not the intention of this paper to delve into the details of risk-neutral valuation, nor is it our belief that we can do a better job than Cox, Ross, and Rubinstein in their famous 1979 paper on option pricing. However, for those still having doubts about risk-neutral valuation, consider this question: How do a buyer and a seller with different risk aversions agree on a price of the same option? If one believes in a market with no arbitrage opportunity, then it must be true that the price of the option is unaffected by the degrees of risk aversion, and therefore it must also be true that the price of the option can be determined by assuming no risk aversion of the investors.

Theory aside, a company writing GLB riders should realize that the result from risk-neutral valuation is the cost it would incur if the company is to transfer the risk to the market. Hedging essentially transfers the risk to the capital market, and the financial assets involved in hedging are priced on the risk-neutral measure. Reinsurance transfers the risk to the reinsurer, and risk-neutral valuation is typically how a reinsurer would charge its reinsurance premium. It would therefore make sense that the company charges a rider fee determined in the same manner.

Unlike most financial options that are short-term and tradable, GLB has much longer coverage periods and cannot be traded. This presents a technical challenge in applying the risk-neutral theory to GLB pricing, because there are typically no observable market inputs for very long dated options. This thus raises an interesting question. Is it appropriate to determine the GLB charge (a long-term source of revenue) based on current market conditions? For example, the financial crisis in 2008 witnessed a dive of the risk-free rates and a soar of the implied volatilities. However, the market condition in the financial crisis was not to persist, and in hindsight it did stop in a relatively short time span. Was it excessive then to levy a GLB charge based on the market condition during the financial crisis?

One way to overcome the problem is to offer a GLB in a successive series of much shorter terms, each having its own price based on the market condition when that piece is offered. In practice, however, such structure could perhaps only be feasible with a guaranteed death benefit rider in a yearly renewable term format. A more realistic scenario, though unlikely to be favored by the industry, is to have the prices of GLB reset on a frequent basis. The prices would then fluctuate over time as market conditions change. The average of these prices would, however, probably be what the insurer should charge by offering the GLB as a single piece with a longer term. It is thus our view that the GLB charge should be assessed with a long-term view, even though this long-term view may be interpreted in different ways. Some may interpret it by ignoring the current market conditions completely, and others by starting at the current market condition and grading over time to the normal condition.

The risk-neutral valuation of the GLB rider is just a first step in the VA pricing process. Ideally, one would want to set the GLB charge at the exact level of the risk-neutral cost. In reality, this step serves as an indication of how expensive the GLB is and it takes a complete profit analysis to eventually determine the adequacy of the GLB charge.

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SECTION IV: PROFIT ANALYSIS

The fundamental objective behind any pricing exercise is to ensure that the new product meets the profit target. By adding a new GLB rider to an existing base, a VA product alters the profit pattern of the latter. First, the GLB charge is a drag on the account value, reducing other asset-based charges. Second, policyholder behavior (e.g., lapses, withdrawals) tends to vary widely, making projections of revenues and losses difficult. Third, the reserve and capital requirement is much more strenuous on a VA contract with the GLB rider. Hence, actuaries have to perform profit analysis on the entire VA product in addition to just studying the GLB itself.

The profitability of the base VA product is driven primarily by the performance of the underlying investments. Without the GLB rider, profit analysis is mostly about finding fees that are adequate to cover expenses and death claims; the fixed expenses (e.g., maintenance) are usually offset directly by fixed charges (e.g., policy fees) and variable expenses offset by asset-based charges. There is as limited a downside loss as an upside gain because the product passes most of the investment risk to the policyholders. Exceptions are perhaps failure to fully recover the acquisition costs or, in the case of a fixed death benefit payment, usually in the form of return of premiums, where the potential death claims may not be fully absorbed by the charges.

The introduction of the GLB, however, substantially increases the risk to the company. The risk of making large sums of guaranteed payments increases exponentially with no matching increase in the upside gain. Therefore, if one were to plot the distribution of the profitability of the VA product with a GLB rider, it would most likely be skewed to the left with a long fat tail. The profitability analysis of a VA product, therefore, should focus not only on the expected average profits but also on the probability of not achieving the target profit or the probability of making a loss. A stochastic process that studies the profit across multiple economic scenarios is necessary.

The profitability analysis of a VA product, therefore, should focus not only on the expected average profits but also on the probability of not achieving the target profit or the probability of making a loss. A stochastic process that studies the profit across multiple economic scenarios is necessary.

The company's reporting framework typically dictates whether the stochastic profit analysis is performed on a risk-neutral or real-world platform. The U.S. statutory reporting framework is on a real-world platform and therefore real-world scenarios fit in the profit analysis stage. In a market-consistent reporting framework, risk-neutral scenarios would be used.

The same type of profit measures as in a non-VA profit study can be used to study VA, such as internal rate of return (IRR), return on assets (ROA), premium margin, etc. Statistical tools, such as histograms or box-whisker charts, can be used to plot the distribution of these measures across multiple economic scenarios. Percentiles, particularly on the left (loss) tail, should be highlighted and reported. Critical p-values, such as probability of losses, should be clearly defined and reported. IRR can sometimes be indeterminable at certain scenarios, and therefore its distribution should be used with caution.

SECTION V: RESERVE AND CAPITAL

Different reporting frameworks may have different reserve and capital requirements. Our report focuses on the U.S. statutory requirements. The left skewness associated with the GLB rider exposes a company to significant potential losses, and the statutory reserve and capital requirements in the United States were established to address this risk.

The statutory reserve, commonly known as AG 43 reserve, requires a projection of surplus of the entire VA contract over multiple real-world economic scenarios. For each scenario, the greatest present value of the accumulated pre-tax deficiencies is calculated. The reserve is equal to the numerical average of the 30% of the largest values of the scenario's greatest present values, or CTE 70. In addition, the reserve is subject to a floor known as the *standard scenario reserve*, which is equal to the greatest present value of the negative *accumulated net revenues* projected with prescribed actuarial assumptions over a deterministic economic scenario of prescribed immediate shock size and recovery rate.

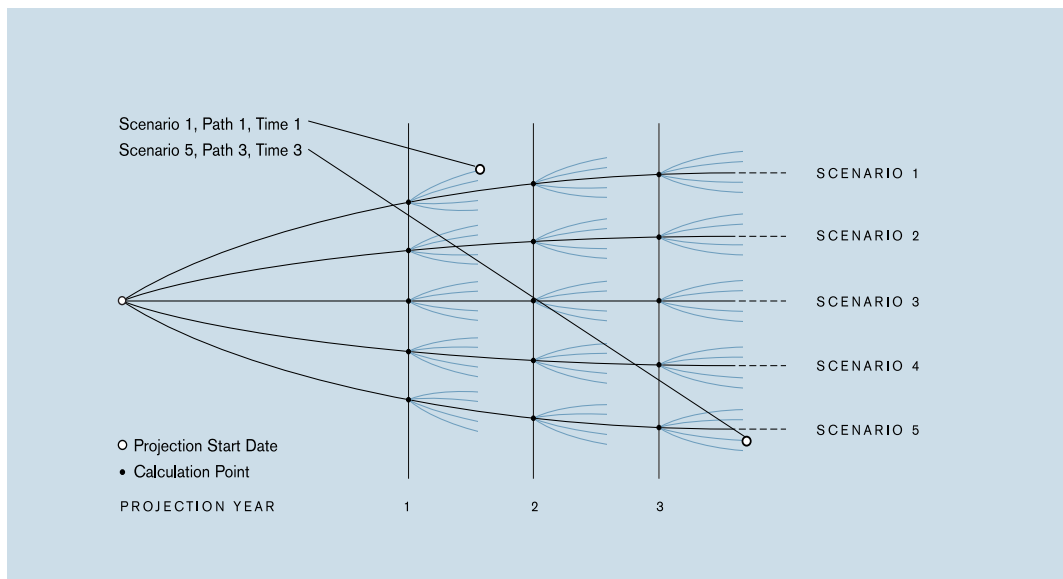
The statutory capital, commonly known as C-3 Phase II capital, uses a similar methodology. Post-tax deficiencies are projected and the capital is equal to the numerical average of the 10% of the largest values of the scenario's greatest present values, or CTE 90. There is also a *standard scenario amount* that acts as a floor for the capital with different prescribed actuarial assumptions and an economic scenario from the reserve requirement.

In the context of a pricing exercise, the profitability is affected by interest earned on the reserve and capital as well as by changes in the reserve and capital. The reserve and capital must be projected over the life of the policy for such impact to be reflected. As discussed in the prior section, the profitability of the VA has to be studied across stochastic economic scenarios. Because determination of the reserve and capital also requires stochastic projections, the profit study of the VA product becomes a nested stochastic process.

Figure 3 depicts a generic nested stochastic process.

Because determination of the reserve and capital also requires stochastic projections, the profit study of the VA product becomes a nested stochastic process.

FIGURE 3: NESTED STOCHASTIC PROCESS



The scenarios in Figure 3 are the outer-loop economic scenarios that determine the distribution of the achieved statutory profits. The calculation points in Figure 3 correspond to the projection cycles in the profit study at which the reserve and capital are calculated. The projection cycles can be in months, quarters, or years, corresponding to the projection frequency of the profit study. The paths in Figure 3 are the inner-loop economic paths for determining the AG 43 reserve and C-3 Phase II capital. These paths are real world with calibration requirements specified by the reserve and capital guidelines. It should be noted that the inner-loop paths may also include a deterministic path for the standard scenario calculation.

In cases where the outer-loop scenarios are also real world, the same real-world scenarios can be applied to both inner loop and outer loop. However, the real-world scenarios for the reserves and capital are subject to prescribed calibration requirements, which may tend to overly stress the left tail. It is thus possible that a different calibration requirement representing a more realistic balance between the left and right tail might be desired for the outer-loop scenarios. It is also possible that the company would develop outer-loop scenarios based on internal economic capital requirements, whereas inner-loop scenarios would be based on regulatory capital requirements.

Typically, a stochastic projection in the United States involves 1,000 scenarios, but companies should perform tests to validate that 1,000 scenarios are indeed sufficient. A nested stochastic process that involves 1,000 outer-loop scenarios and 1,000 inner-loop paths could take days or even weeks of run time on a single computer. A balance needs to be drawn between practicality and accuracy. However, actuaries have to be cautious about reducing the number of inner-loop paths. The reserve is determined by 30% of the total inner-loop paths, and capital by 10%. A small number of inner-loop paths may not generate a large enough subset in the two tails to produce reasonable reserve and capital results.

As an alternative to a nested stochastic process, it is possible to use a table approach. The table approach establishes a table that stores the reserve and capital factors varied by dimensions such as in-the-moneyness (ITM) of guarantee, age/sex, and in-force duration. These factors, typically ratios of reserve/capital amounts over account value, are generated before the profit test and then loaded as inputs in the profit test exercise. The factors may also explicitly consider the floor of the standard scenario calculation. The generation calls for multiple stochastic projections, the exact number of which depends on the combinations of dimensions in the table. In essence, a nested stochastic process is transformed to multiple stochastic processes. The generation of the table is not trivial work, but the result can provide a very useful basis on which to extrapolate the impact of reserves and capital on different business mixes at different economic conditions. The projected reserve and capital amounts will then be the projected account values multiplied by the corresponding ratios based on the combination of ITM ratio and in-force duration.

SECTION VI: IMPACT OF HEDGING

As one of the most common and effective risk management tools, hedging affects the profitability of the VA product in different ways.

Hedging introduces additional costs from setting up and maintaining the hedging platform as well as transacting the hedge assets. On average, hedging is expected to reduce the expected profits of the VA product.

Hedge assets are typically derivatives whose price sensitivities to different market factors match those of the GLB. With an appropriate trade position, the value of the hedge assets would move in the opposite direction to the value of the GLB guarantees. Consequently, hedging is expected to produce gains in bad economic scenarios and losses in good economic scenarios.

The combination of additional costs and potential gains or losses affects the AG 43 reserve and C-3 Phase II capital amounts. Though expected to reduce the reserve and capital level, the exact impact of hedging is not straightforward and may vary depending on the exact hedging strategy.

The typical hedging strategy in the United States is to dynamically hedge the change in the fair value of the guaranteed claims against different movements in the market factors. The fair value of the guaranteed claims is assessed on a risk-neutral basis. The market factors typically include equity index levels, interest rates, and equity-implied volatilities. More specifically, the sensitivity to these factors is commonly known as the "Greeks" in the finance area: delta, rho, and vega. Hedging is thus essentially targeted to dynamically hedge one or more Greeks by transacting hedge assets to produce Greeks of the opposite sign.

The dynamic Greek hedging, however, brings an interesting problem in U.S. statutory reporting. Measured on a risk-neutral basis, dynamic Greek hedging is very effective in eliminating earnings volatility on a market-consistent basis. However, the impact of such hedging on the AG 43 reserve and C-3 Phase II capital is much less certain.

There are several disconnects between the dynamic Greek hedging and the statutory reserve and capital requirements. First, the Greeks are measured as changes in the fair values of the guaranteed claims, which are assessed with risk-neutral scenarios. The statutory reserve and the capital are assessed with the tails of real-world scenarios. The risk-neutral scenarios are calibrated to the current market prices of derivatives and practically change every day. On the other hand, real-world scenarios are calibrated to historical movements over years and thus may not change dramatically. The interest rates for discounting are also generated in different ways.

Second, the Greek hedging focuses on the value of the GLB rider only, but the statutory reserve and capital takes into consideration cash flows of the entire contract. The cash flows of the non-GLB piece are typically much less volatile than those of the GLB, and might lower the sensitivity of the reserve and capital to market changes.

Third, both reserve and capital guidelines have specific restrictions on the extent to which the hedging effectiveness can be reflected in the calculations. In the case of standard scenario, the impact of dynamic hedging is not allowed in the reserve and capital calculations. This makes assessing the impact of hedging on reserves and capitals murkier.

The uncertainty of the hedging impact makes it critical that hedging be studied carefully in the pricing exercise. However, hedge modeling is complicated, and requires certain unique skills that have not been part of the traditional actuarial training. Actuaries would need to work closely with the investment team or the hedge team to correctly model hedging activity.

With an appropriate trade position, the value of the hedge assets would move in the opposite direction to the value of the GLB guarantees. Consequently, hedging is expected to produce gains in bad economic scenarios and losses in good economic scenarios.

Perhaps the easiest way to model hedging is similar to the way that reinsurance is often modeled. A hedge effectiveness ratio of $x\%$ is assumed. Hedge modeling would result in a cash income to the insurer of an amount equal to $x\%$ of the GLB claim amounts. Sometimes the hedge effectiveness might be assumed to be $x\%$ of the change in fair values of the GLB. On the other hand, there is an additional cash outgo from the insurer that represents the cost of hedging.

This is perhaps the simplest way to model hedging, but it does not realistically reflect the impact. The method might work reasonably well when analyzing the average result of the multiple scenarios. However, it is likely to misstate the impact of hedging when a specific scenario is studied. In other words, a hedge effectiveness ratio of $x\%$ on average does not imply $x\%$ effectiveness for a particular scenario. Actuaries need to consult investment or hedging experts on what would be a rational assumption of $x\%$ and also the hedging cost.

It is possible to simulate the dynamic Greek hedging in a more accurate way. This typically requires a nested stochastic projection to calculate the Greeks of the GLB liabilities at every calculation point along each outer-loop scenario as illustrated in Figure 3. It also requires modeling the transaction of derivative assets to match the Greeks of the GLB liabilities throughout the projection. The hedging cost and income would then vary at every projection point and every outer-loop scenario, thus producing a more accurate impact of the hedging strategy.

However, this accuracy should not be overestimated. No simulation is likely to truly reflect the hedging in real life, which typically rebalances hedging positions on a much more frequent basis than assumed in a pricing exercise. Weekly or even daily rebalancing in real life is common, but a pricing exercise hardly ever projects more frequently than monthly. The run time required to perform dynamic hedging simulation can be prohibitive, and therefore simplifications in the modeling are probably inevitable, leading to loss of accuracy.

It is important that actuaries consult investment and hedging experts about the modeling methodology and the assumptions. It is also imperative to recognize the limitation of the methodology and achieve a balance between accuracy and practicality.

Regardless of the approach adopted in hedge modeling, it is important that actuaries consult investment and hedging experts about the modeling methodology and the assumptions. It is also imperative to recognize the limitation of the methodology and achieve a balance between accuracy and practicality.

SECTION VII: SUMMARY

The four elements we have discussed so far represent four major analyses of a comprehensive VA pricing process. These analyses may not necessarily be performed in any particular order, and may be repeated to search for the optimal results. We have discussed some of the general considerations involved in these analyses, and have included more detailed discussions on some of the technical challenges in the Appendix. However, the nature of the VA pricing is so complicated that it is impossible to touch upon all that needs to be considered. In the following paragraphs, we will discuss briefly two other important considerations.

First, the VA pricing results are sensitive to dynamic policyholder behavior assumptions. The option nature of the GLB in VA offers policyholders opportunities to exercise against the company. Thus, the fundamental question behind any dynamic policyholder behavior assumption is whether policyholders act rationally and always exercise at the optimal time. The historical experience is unfortunately still scant because of the relatively short history of GLB products in the market. Therefore, it is a challenge to form a realistic policyholder behavior assumption. Where there is little reliance on data to form the dynamic formula, it is then crucial for actuaries to perform adequate sensitivity tests to study the results.

Second, VA products now impose more asset allocation restrictions than prior to the financial crisis. Some of the big VA writers have introduced structured asset allocation programs that automatically allocate investments between risky and less risky funds based on prescribed mathematical models. In some cases, such allocation programs may create specific marketing appeal because they might potentially allow more investments in risky funds than traditional allocation programs. It is thus important for actuaries to be able to reflect the structured asset allocation program in the pricing process.

As a summary of this report, here is a list of its key points:

- The adequacy of the GLB charge should be assessed by performing risk-neutral valuation.
- The profitability of the entire contract should be analyzed through a stochastic projection with focus on both mean and tail results.
- The impact of reserve and capital requirements should be analyzed in the profit analysis.
- The impact of hedging should be analyzed in the reserve and capital projections as well as the profit analysis.
- Ultimately, the most desired goal of a VA pricing is a product that attracts sales and brings in reasonable profitability by accepting a well-calculated risk.

Ultimately, the most desired goal of a VA pricing is a product that attracts sales and brings in reasonable profitability by accepting a well-calculated risk.

The case study focuses on discussions of the technical modeling challenges.

APPENDIX: CASE STUDY

In this Appendix, we will study in more detail some of the technical challenges in VA pricing. To facilitate the discussion, we will use a case study to present some illustrative calculations. It should be emphasized that the case study focuses on discussions of the technical modeling challenges instead of the results. Also, in some cases, there may not be a best answer to a particular technical challenge and we will thus provide different approaches. We will attempt to be impartial in discussing these approaches, as the ultimate choice among them may depend on specific circumstances.

The case study showcases a sample VA policy with a typical guaranteed living withdrawal benefit (GLWB) rider. The table in Figure 4 summarizes the policy specifications.

FIGURE 4: SAMPLE POLICY SPECIFICATIONS	
ISSUE AGE/SEX	55/FEMALE
FIRST WITHDRAWAL	BEGINNING OF POLICY YEAR 6
BENEFIT BASE	MAXIMUM OF 5% COMPOUND ROLL-UP AND ANNUAL RATCHET FOR UP TO YEAR 5, AND ANNUAL RATCHET THEREAFTER
CHARGE AS A PERCENT OF BENEFIT BASE	0.6%
MAXIMUM BENEFIT PERCENT	5% "FOR LIFE"
FUND ALLOCATION	100% IN S&P500

The fund allocation of 100% in equity fund is a simplification in our case study. In reality, restrictions in risky investments are crucial in managing the risk of a GLWB product.

Risk-neutral valuation

In Section III we discussed that the rider charge should be assessed on a risk-neutral basis. We also discussed whether the risk-neutral cost should be calculated on a current market basis or allowing for a long-term view. The table in Figure 5 shows the risk-neutral cost of the GLWB under different assumptions.

FIGURE 5: RISK-NEUTRAL COST, 12/31/2009	
RISK-NEUTRAL COST (AS PERCENT OF BENEFIT BASE)	RISK-NEUTRAL SCENARIOS
1.7%	RISK-FREE RATE: SWAP CURVE OF 12/31/2009 EQUITY VOLATILITY: MARKET IMPLIED 12/31/2009 IN FIRST FIVE YEARS, GRADE DOWN TO 17.35% FROM YEARS 5 TO 15, STAY AT 17.35% AFTER YEAR 15
1.9%	RISK-FREE RATE: SWAP CURVE OF 12/31/2009 EQUITY VOLATILITY: MARKET IMPLIED 12/31/2009 IN FIRST FIVE YEARS, STAY LEVEL AFTER YEAR 5
1.5%	RISK-FREE RATE: 4% EQUITY VOLATILITY: 17.5%

For comparison purposes, Figure 5 shows the risk-neutral costs corresponding to different risk-neutral scenarios. The cost would increase slightly if we remove the grading of equity volatility and would reduce slightly if we assume long-term equity volatility throughout. The risk-neutral costs with the long-term market assumption are not much lower because the market-implied volatilities as of December 31, 2009, were not very much above the long-term assumption. Had we calculated this with market-implied volatilities during the financial crisis, the difference would be much more significant. The table in Figure 6 shows the risk-neutral costs corresponding to market conditions as of December 31, 2008.

FIGURE 6: RISK-NEUTRAL COST, 12/31/2008

RISK-NEUTRAL COST (AS PERCENT OF BENEFIT BASE)	RISK-NEUTRAL SCENARIOS
3.5%	RISK-FREE RATE: SWAP CURVE OF 12/31/2008 EQUITY VOLATILITY: MARKET IMPLIED 12/31/2008 IN FIRST FIVE YEARS, GRADE DOWN TO 17.35% FROM YEARS 5 TO 15, STAY AT 17.35% AFTER YEAR 15
3.7%	RISK-FREE RATE: SWAP CURVE OF 12/31/2008 EQUITY VOLATILITY: MARKET IMPLIED 12/31/2008 IN FIRST FIVE YEARS, STAY LEVEL AFTER YEAR 5

Figure 6 shows that the risk-neutral cost with December 2008 market conditions was more than double that with the long-term market assumption. Both Figures 5 and 6 also show that the cost is not very different if grading is applied. The claims under the GLB product do not occur until the policy account value is depleted. The time of depletion depends on both withdrawal activity and the underlying asset performance. Under the risk-neutral projection, this most often occurred between years 10 and 15, at which the spot volatility produced by the grading approach was not far different than the no-grading approach. The cost might be more different if the grading were over a shorter period.

The sample policy has a rider charge of 0.6%. However, results in both tables suggest that the true cost of the GLWB guarantee is considerably higher than 0.6%, partly because we assumed 100% investments in equity. Raising the charge to the rates in Figure 6 is probably excessive because the product offers a long-term guarantee and the extreme market condition as of December 31, 2008, was temporary. Consequently, the rates in Figure 5 are probably more representative of a reasonable long-term cost of the guarantee, assuming products sold over time.

Reserves and Capital

In Section V, we discussed two different approaches for projecting the AG 43 reserve and C-3 Phase II capital in a pricing exercise.

A nested stochastic projection has a high requirement on both the projection software and the modeling skill. A table approach probably requires little modeling effort, but requires much more effort in setting up all the pre-calculation runs as well as in summarizing the vast amount of run results.

The table in Figure 7 compares the results from the nested stochastic projection and the table approach. We performed the comparison on a scenario with annual flat equity return of 8% and a scenario with annual flat equity return of -5%. In both scenarios, we assumed that the yield curves stay unchanged from the valuation date.

The rates in Figure 6 is probably excessive because the product offers a long-term guarantee and the extreme market condition as of December 31, 2008, was temporary. Consequently, the rates in Figure 5 are probably more representative of the true long-term cost of the guarantee.

Hedging

In the case study, we modeled hedging in three different ways. We have seen all three approaches applied by different companies in the United States, with the first one perhaps being the most common thanks to its simplicity. The last approach is the most complicated and is probably the closest to actual hedging in reality, but it is still an approximation and its limitations should be duly noted.

The first approach is a *recovery of claim* approach, where hedge benefits are modeled as 90% of the GLWB claims and hedge costs as 1.7% of the projected guaranteed GLWB base amounts. The 90% is an assumed hedge effectiveness ratio, and is selected in our case to suggest a rather effective hedge program. In reality, actuaries should consult the hedge team for a reasonable assumption. The 1.7% is the hedge cost shown in Figure 5.

The second approach is a *recovery of fair value* approach, where hedge benefits are modeled as 90% of the changes in the fair values of the GLWB benefits and hedge costs still as 1.7% of the projected guaranteed GLWB base amounts. The first approach only produces a hedge benefit when there is a claim, but in reality hedge cash flows occur as hedge transactions take place. The *recovery of fair value* approach explicitly acknowledges that a hedge program is typically structured to hedge the changes in the fair values of the guarantee, which are determined as the average present value of GLWB claims less average present value of GLWB revenues under risk-neutral scenarios. Instead of simulating hedge transactions, the approach simulates the ultimate result of hedging, which is the degree to which the liability fair value reserve is matched. The approach apparently requires projection of future fair values, which can be accomplished through either a nested stochastic simulation or a table approach similar to that described in Section V.

The third approach is a *delta simulation* approach, where dynamic hedge is simulated to match the delta of the GLWB benefits. Delta measures the changes in the fair values of GLWB guarantees, again calculated similar to a FAS 133 valuation, per 1% change in the underlying equity investments. Companies that hedge would typically hedge delta. Many companies hedge rho as well, which measures change in fair value per change in interest rates. Fewer companies hedge all the Greeks, which would be too expensive. Hedge assets are modeled as one-month equity futures contracts, which are rebalanced at every month end. In reality, equity futures are rebalanced more frequently and incur costs from rebalancing. We made no explicit assumptions as to the hedge effectiveness or hedge costs, which essentially varied across scenarios.

In both second and third approach, the inner paths, as depicted in Figure 3, are the same risk-neutral scenarios as those resulting in the 1.7% hedge cost in Figure 5. However, to reduce run time, we compressed the 1,000 risk-neutral paths to 100 paths. The compression was performed by assigning the 1,000 scenarios into 100 clusters where the assignment was based on how distant each scenario is compared to each of the 100 clusters. This resulted in the loss of a certain degree of accuracy, where the hedge cost as a result of the compressed 100 paths was 1.8% instead of 1.7%.

In both the second and third approaches, monthly projection was adopted, which is probably the best a pricing projection could achieve in terms of balance between accuracy and practicality. In reality, hedge transactions may have to be made on daily or weekly bases.

The third approach simulates delta hedge only. The effectiveness of delta-only hedge may be lower than the 90% effectiveness assumed in both the first and second approaches, particularly in a volatile scenario.

With all three approaches, hedge modeling introduces additional cash flows to the pricing projection and correspondingly results in different levels of statutory reserves and capital. The second and third approaches both require nested stochastic projections and may thus require a nested-within-a-nested stochastic projection if they were to be built in the reserve and capital projection. The modeling would get too complicated and run time would be prohibitively long. Also, as hedging affects both cash flows and the reserve/capital levels, a projection with layers of nested stochastic projections makes it difficult to determine the different impacts of hedging. Therefore, it would make sense to replace one of the nested stochastic projections with an alternate approach. We will close our case study by studying the impact of

In the case study, we modeled hedging in three different ways.

hedging on cash flows and reserve/capital levels separately. We will illustrate the results using each of the three hedge modeling approaches described, but our illustration is purely for discussion and it remains the responsibility of each actuary to select the approach appropriate for each set of circumstances.

Impact on Cash Flows

The table in Figure 8 illustrates the impact of hedging on cash flows under two deterministic scenarios.

FIGURE 8: IMPACT OF HEDGING ON CASH FLOWS: DETERMINISTIC

SCENARIO 1 - LEVEL 8% EQUITY RETURN	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6	YEAR 7	YEAR 8	YEAR 9	YEAR 10
HEDGE NET CASH FLOWS										
RECOVERY OF CLAIM	-100	-103	-105	-106	-107	-105	-98	-60	-54	-48
RECOVERY OF FAIR VALUE	-10	-68	-79	-93	-109	-122	-119	-95	-83	-72
DELTA SIMULATION	-70	-74	-63	-59	-56	-54	-49	-43	-36	-32
BENEFIT / AV	100%	100%	100%	99%	99%	100%	100%	100%	100%	100%
SCENARIO 2 - LEVEL -5% EQUITY RETURN	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6	YEAR 7	YEAR 8	YEAR 9	YEAR 10
HEDGE NET CASH FLOWS										
RECOVERY OF CLAIM	-100	-103	-105	-108	-111	-112	-111	-77	-77	-76
RECOVERY OF FAIR VALUE	117	67	66	97	140	163	166	175	150	139
DELTA SIMULATION	69	81	89	98	116	124	99	84	70	48
BENEFIT / AV	114%	129%	147%	168%	191%	229%	284%	366%	500%	758%

Figure 8 compares the hedge net cash flows, defined as hedge benefits less hedge costs, of the different hedge modeling approaches under a level 8% equity return scenario and a level -5% equity return scenario. Under the latter, the account value crashes in year 13 and the claim is captured as a lump sum representing a present value of future GLWB payments. Cash flows in Figure 8 are normalized such that -100 equals the first-year cash flow under the *recovery of claim* approach.

Under the level 8% scenario, the hedge net cash flows are negative with all three approaches. This makes sense because hedge losses are expected under good equity scenarios where there are no projected claims. With the *recovery of claim* approach, hedge net cash flows are simply the hedge cost of 1.7% multiplied by projected GLWB benefit base amounts.

With the *recovery of fair value* approach, hedge net cash flows are higher than the *recovery of claim* approach when there are increases in fair values of GLWB and lower when there are decreases in the fair values. Results show that the fair values increase in the first four years followed by continual decreases. The increases in early years should result from the roll-up feature of the GLWB making the guaranteed base amounts outgrow the account values. Roll-up stops after year 5 and thus account value starts to pick up quickly following a strong positive equity return.

With the *delta simulation* approach, hedge net cash flows are consistently higher than the *recovery of claim* approach partly because the hedge costs are lower than the constant 1.7% assumed in the latter. As account values grow under a strong positive scenario, hedge costs are expected to decline over time as options grow out-of-the-money.

Under the level -5% scenario, the hedge net cash flows are negative with the first approach and positive under the other two. However, not shown in Figure 8, with the first approach there is a big positive hedge cash flow in the year of claims whereas with the other two approaches there is no big increase. To a certain extent, the second and third approaches spread the ultimate hedge benefit over the projection period. The *delta simulation* approach produces consistently lower hedge net cash flows than the *recovery of fair value* approach because delta-only hedging should produce less than 90% effectiveness.

Figure 8 is useful to validate and rationalize the hedge modeling, but the real impact of hedging on expected net cash flows has to be revealed through stochastic projections. The table in Figure 9 shows the impact of hedging on cash flows across 1,000 real-world scenarios. We used the same real-world scenarios as those for the reserve and capital calculations, which does not have to be the case in reality.

The real impact of hedging on expected net cash flows has to be revealed through stochastic projections.

FIGURE 9: IMPACT OF HEDGING ON CASH FLOWS: STOCHASTIC

	NO HEDGE	RECOVERY OF CLAIM	RECOVERY OF FAIR VALUE	DELTA SIMULATION
MEAN	0.60%	-1.14%	-0.56%	-0.69%
MAXIMUM	1.43%	-0.17%	1.97%	1.30%
MINIMUM	-7.10%	-5.51%	-1.02%	-2.54%
FIRST PERCENTILE	-3.6%	-3.9%	-1.0%	-1.6%
FIFTH PERCENTILE	-0.95%	-2.58%	-0.96%	-1.15%
STANDARD DEVIATION	0.89%	0.71%	0.37%	0.34%

Figure 9 shows the statistics of the ratios of present values of net cash flows divided by present values of projected account values. Both present values are discounted at 12%, our assumed target profit rate consistent with Figure 7. The results would of course be different at a different discount rate, particularly in the case of the *recovery of claim* approach, where large positive hedge payments occur in the year of claim.

All three hedge modeling approaches produce lower standard deviations than no hedge at all, but also produce lower mean results. All three approaches also manage to improve the results of the worst scenario. Despite the differences in details, all three approaches suggest that the hedge modeling produces better results under the left tails, or bad economic scenarios, at a cost that turns average expected net cash flows from positive to negative. The high cost is consistent with results in Figure 5, where hedge cost is much higher than the rider charge, suggesting that the GLWB rider might be underpriced. Would the company then be better off not hedging? Remember the year of 2008? Tail events can and did occur.

Figure 10 plots the ratios of present values of net cash flows divided by present values of projected account values across the 1,000 scenarios of each modeling.

FIGURE 10: IMPACT OF HEDGING ON NET CASH FLOWS: STOCHASTIC

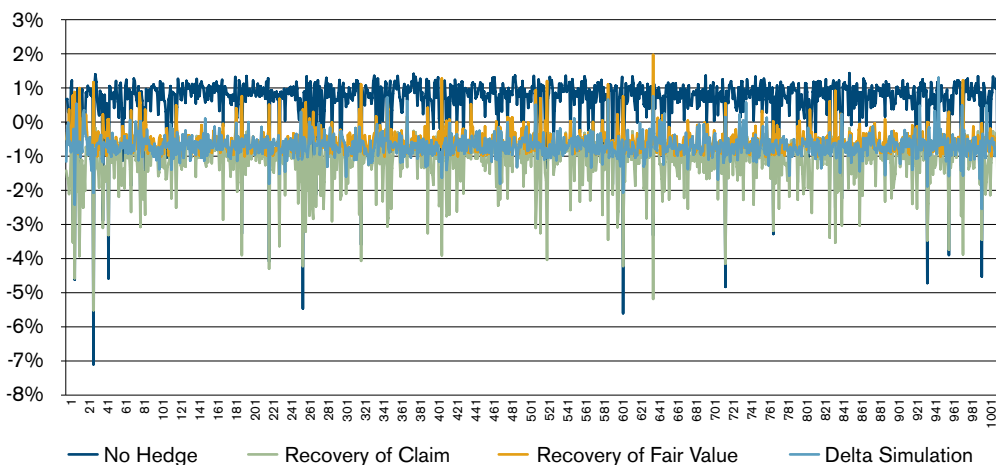


Figure 10 shows that some of the ratios of no hedging drop deep down toward the bottom of the chart. The *recovery of claim* results also have some ratios stretching down, but not as far as the results from not hedging at all. Both the *recovery of fair value* and *delta simulation* results show much more centralized clusters consistent with the lower standard deviation in Figure 9.

Impact on AG 43 and C-3 Phase II

We will focus on the stochastic results of the AG 43 and C-3 Phase II because the standard scenario calculation does not give credit to dynamic hedging. However, one can still allow for the hedge assets held at the valuation date in the standard scenario calculation, which would typically be reasonably effective at protecting against the equity shocks prescribed under the standard scenario.

Recall that both stochastic AG 43 reserve and stochastic C-3 Phase II capital aim to measure the accumulated negative surplus at the left tails of the real-world scenarios, with the main differences in the CTE level and tax. The answer to whether or not hedging would reduce the stochastic reserve or capital really lies in whether the hedge benefit at the corresponding CTE level would outweigh the associated hedge cost. The analysis in Figure 9 may shed some light on this. Figure 9 shows the various statistics of the ratios of present values of net cash flows divided by present values of projected account values, but it does not show the CTE measures. The table in Figure 11 shows the CTE70 and CTE90 of the ratios from the same analysis.

FIGURE 11: CTE MEASURES OF NET CASH FLOWS

	NO HEDGE	RECOVERY OF CLAIM	RECOVERY OF FAIR VALUE	DELTA SIMULATION
CTE70	-0.24%	-2.0%	-0.88%	-1.04%
CTE90	-1.54%	-2.8%	-0.96%	-1.27%

Both the *recovery of fair value* and *delta simulation* approaches produce worse CTE70 results than not hedging at all, and better CTE90 results. In other words, in present value terms, both modeling approaches suggest that the hedge benefit does not outweigh the hedge cost at CTE 70 level but does so at CTE 90 level. Even though the present value terms are not exactly what AG 43 and C-3 Phase II look at, this might still be an indication that hedging may produce higher AG 43 reserves but lower capital requirements.

The table in Figure 12 compares the AG 43 stochastic reserve levels with and without hedging. For illustration purpose, we selected the *delta simulation* approach to represent with hedging.

FIGURE 12: AG 43 WITH HEDGING VS. WITHOUT HEDGING

WITH HEDGING									
ITM/YEAR	150	130	100	90	80	70	60	50	20
1	12.5%	11.8%	14.6%	17.6%	22.4%	30.8%	42.4%	57.9%	114.3%
2	12.5%	11.6%	13.4%	16.2%	20.9%	29.7%	41.7%	56.2%	109.6%
3	13.3%	12.4%	13.1%	15.8%	21.1%	29.8%	42.4%	56.7%	108.0%
4	13.1%	12.1%	11.7%	14.2%	19.3%	28.1%	40.6%	55.5%	103.2%
5	8.7%	7.7%	6.3%	8.4%	12.4%	20.1%	32.3%	47.6%	92.6%
10	8.8%	7.8%	6.4%	8.9%	13.1%	22.4%	37.3%	56.4%	115.8%
WITHOUT HEDGING									
ITM/YEAR	150	130	100	90	80	70	60	50	20
1	6.6%	6.1%	7.8%	10.4%	14.7%	21.0%	29.4%	38.7%	69.8%
2	6.8%	6.3%	7.4%	10.0%	14.0%	20.2%	28.5%	38.0%	66.3%
3	7.4%	7.3%	8.2%	10.4%	14.1%	20.2%	28.6%	37.8%	63.5%
4	8.4%	8.0%	8.0%	9.9%	13.2%	19.1%	27.5%	36.7%	60.7%
5	5.4%	4.8%	3.9%	5.4%	8.5%	14.2%	22.4%	31.3%	53.7%
10	4.9%	4.3%	3.7%	5.4%	8.9%	15.2%	25.5%	37.0%	66.5%

The rows in Figure 12 represent the number of years for which the policy is in force. The columns represent the different ITM levels, with ITM defined as account value divided by GLWB base amounts. Therefore, the row of 5 and column of 90 produces the AG 43 reserve result for a policy that is in force at end of policy year 5 with the then account value of 90% of the GLWB base amount. The reserve is the AG 43 requirement in addition to the cash surrender value and expressed as percentages of the initial deposit. Figure 12 confirms our conjecture that hedge modeling results in higher AG 43 reserve levels.

The reserve is the AG 43 requirement in addition to the cash surrender value and expressed as percentages of the initial deposit.

The table in Figure 13 compares the C-3 Phase II stochastic capital levels with and without hedging.

FIGURE 13: C-3 PHASE II WITH HEDGING VS. WITHOUT HEDGING									
WITH HEDGING									
ITM/YEAR	150	130	100	90	80	70	60	50	20
1	16.4%	14.9%	15.9%	18.2%	21.3%	25.4%	29.6%	34.8%	55.2%
2	15.6%	14.1%	14.5%	16.5%	19.6%	23.5%	27.8%	33.1%	51.4%
3	15.4%	14.0%	13.6%	15.6%	18.6%	22.3%	26.8%	32.0%	47.8%
4	14.9%	13.5%	12.1%	13.9%	16.8%	20.5%	24.9%	30.4%	43.8%
5	11.3%	9.9%	8.0%	9.8%	12.7%	16.3%	20.8%	26.0%	38.2%
10	10.4%	9.2%	7.6%	9.9%	13.6%	18.0%	23.3%	30.1%	46.7%
WITHOUT HEDGING									
ITM/YEAR	150	130	100	90	80	70	60	50	20
1	15.2%	13.9%	15.3%	18.0%	21.8%	25.8%	29.9%	34.2%	55.9%
2	15.3%	13.9%	14.2%	16.8%	20.4%	24.7%	28.6%	33.3%	51.1%
3	15.7%	14.3%	13.8%	16.1%	19.6%	23.8%	28.1%	32.2%	46.6%
4	15.5%	14.0%	12.7%	14.8%	18.1%	22.3%	26.5%	30.6%	42.8%
5	12.1%	10.6%	8.5%	10.5%	13.9%	18.3%	22.4%	26.4%	37.1%
10	11.0%	9.8%	8.3%	11.0%	15.4%	20.8%	26.2%	31.5%	45.5%

The capital is the C-3 Phase II requirement in addition to the cash surrender value and expressed as percentages of the initial deposit.

The capital is the C-3 Phase II requirement in addition to the cash surrender value and expressed as percentages of the initial deposit. It is not capital required on top of the AG 43 reserves, which would be zero when the guarantee is deep in-the-money in this case. Figure 13 shows that hedge modeling results in lower C-3 Phase II capital levels in most of the cells of the table. However, because the AG 43 requirement is higher than the C-3 Phase II requirement, particularly with hedge modeling, hedge modeling overall results in a higher statutory asset requirement.



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